# Mechanical Engineering

**Thermodynamics**
Work
Heat
Energy and th First law of thermodynamics
Properties and State
Ideal gas
Entropy and the Second law of thermodynamics
The thermodynamics of state
Carnot cycle
Introduction to statistical thermodynamics

A simple engine uses an ideal gas as the working fluid in a piston-cylinder system. The gas is first heated at constant pressure from state 1 to state 2, then cooled at constant volume to state 3 where $T_3 = T_1$, and then "cooled" at constant temperature, thereby returning to state 1. Derive expressions for the amounts of energy transfer as work and heat for each process in terms of the temperatures at each state and the constants of the gas.

Assuming $T_1$ = 300K, $p_1$ = 0 .1 MPa, $T_2$ = 700K, and $k = c_p/c_v = 1.4$, calculate the cycle efficiency.

Solution
<u>State 1 to state 2</u>
gas is first heated at constant pressure:
$Q = n\,c_p\,(T_2 - T_1)$
$\Delta U = n\,c_v\,(T_2 - T_1)$
$W = \Delta U - Q = n\,(c_v - c_p)(T_2 - T_1) = -\,nR\,(T_2 - T_1)$

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} = nR$$

$$p_1 = p_2$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

<u>State 2 to state 3</u>
cooled at constant volume:
$Q = n\,c_v\,(T_3 - T_2)$
$\Delta U = n\,c_v\,(T_3 - T_2)$
$W = \Delta U - Q = 0$

$$\frac{p_3V_3}{T_3} = \frac{p_2V_2}{T_2} = nR$$

$$V_3 = V_2$$

$$\frac{p_3}{p_2} = \frac{T_3}{T_2}$$

<u>State 3 to state 1</u>

"cooled" at constant temperature:

$\Delta U = m\, c_v\,(T_1 - T_3) = 0$

$$W = \int_{State3}^{State1} -p\,dV$$

$$p = \frac{nRT}{V}$$

$$W = \int_{State3}^{State1} -\frac{nRT}{V}\,dV = -nRT_1 \int_{State3}^{State1} \frac{dV}{V} = -nRT_1 \ln\frac{V_1}{V_3}$$

$Q = \Delta U - W = -W$

$$Q = nRT_1 \ln\frac{V_1}{V_3}$$

$$\frac{V_1}{V_3} = \frac{V_1}{V_2}\frac{V_2}{V_3} = \frac{T_1}{T_2}$$

$$W = -nRT_1 \ln\frac{T_1}{T_2}$$

$$Q = nRT_1 \ln\frac{T_1}{T_2}$$

<u>Efficiency</u>

$$\eta = \frac{-W_{cycle}}{Q_{cycle}}$$

$$W_{cycle} = -nR\left(T_2 - T_1\right) + 0 - nRT_1 \ln\frac{T_1}{T_2}$$

$$Q_{chauffage} = nc_p\left(T_2 - T_1\right)$$

$$\eta = \frac{-W_{cycle}}{Q_{chauffage}} = \frac{nR\left(T_2 - T_1\right) + nRT_1 \ln\frac{T_1}{T_2}}{nc_p\left(T_2 - T_1\right)}$$

$$c_p = \frac{k}{k-1}R$$

$$\eta = \frac{nR\left(T_2 - T_1\right) + nRT_1 \ln\frac{T_1}{T_2}}{n\frac{k}{k-1}R\left(T_2 - T_1\right)} = \frac{\left(T_2 - T_1\right) + T_1 \ln\frac{T_1}{T_2}}{\frac{k}{k-1}\left(T_2 - T_1\right)}$$

$$\eta = \frac{\left(700 - 300\right) + 300\ln\frac{300}{800}}{\frac{1.4}{0.4}\left(700 - 300\right)} = \frac{105.75}{1400} = 7.55\%$$

Air at $T_1$ = 300K and $p_1$ = 0.3MPa is heated at constant temperature up to $T_2$ = 700K. Determine the change in internal energy of per unit mass, using the law:

$c_p(T) = \alpha + \beta T + \gamma T^2 + \delta T^3$

where the temperature $T$ is in K, and considering the values:
$\alpha$ = 28.11 kJK$^{-1}$kmol$^{-1}$
$\beta$ = 0.00197 kJK$^{-2}$kmol$^{-1}$
$\gamma$ = 0.0000048 kJK$^{-3}$kmol$^{-1}$
$\delta$ = - 2 10$^{-9}$ kJK$^{-4}$kmol$^{-1}$

Solution

$dU = mc_v dT$

$c_v = c_p - R = \alpha - R + \beta T + \gamma T^2 + \delta T^3$

$\Delta U = U_2 - U_1 = \int_{state1}^{state2} \left(\alpha - R + \beta T + \gamma T^2 + \delta T^3\right) dT$

$\Delta U = (\alpha - R)\left(T_2 - T_1\right) + \frac{\beta}{2}\left(T_2^2 - T_1^2\right) + \frac{\gamma}{3}\left(T_2^3 - T_1^3\right) + \frac{\delta}{4}\left(T_2^4 - T_1^4\right)$

$\Delta U = (28.11 - 8.314)\left(700 - 300\right) + \frac{0.00197}{2}\left(700^2 - 300^2\right) + \frac{4.8 \cdot 10^{-6}}{3}\left(700^3 - 300^3\right) - \frac{2 \cdot 10^{-9}}{4}\left(700^4 - 300^4\right)$

So, per unit mass:

$\Delta U_{mass} = \frac{\Delta U}{M} = \frac{8702 kJkg^{-1}}{29 kgkmol^{-1}} = 300 kJkg^{-1}$

**Fluid mechanics**

Hydrostatics
Kinematics of fluids
Dynamics of Newtonian fluids
Conservation of mass, momentum and mechnical energy
Boundary layer, drag, flow over external surfaces
Introduction to gas dynamics, speed of sound, normal shocks

**Heat transfer**

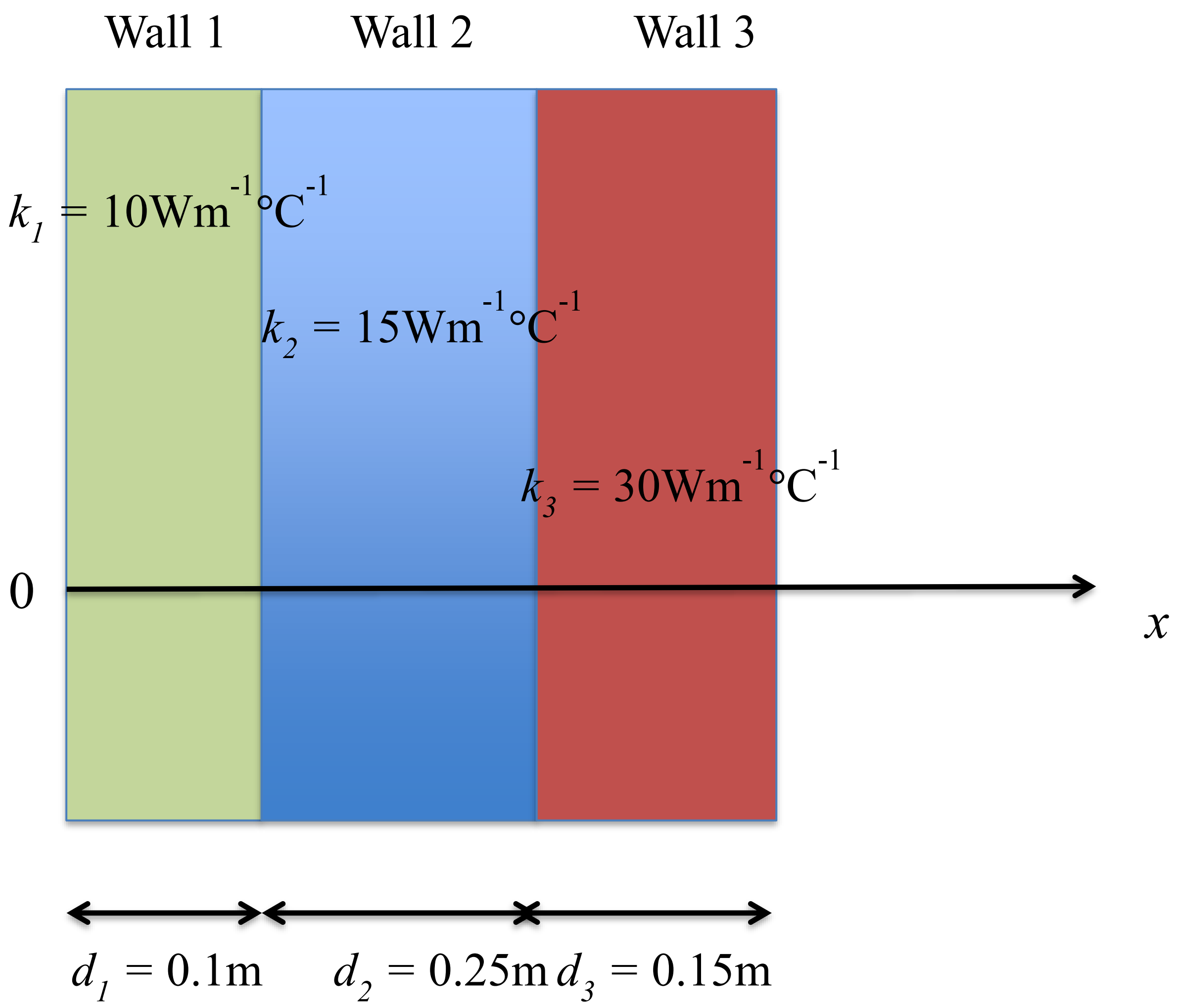
Basic mechanisms of heat transfer
Law of conservation of energy
Conduction
Convection
Radiation

A heat generating wall 2 is sandwiched between two other walls 1 et 3. There is no heat generation in wall 1 or 3. The temperature distribution in wall 2, because of the heat generation, is described by the law:

$T_2(\mathrm{x}) = a + bx + cx^2$

Where $T$ is in °C, and $x$ is the distance from the left side of wall 1, as described on the figure. The wall 1 is in contact with a fluid at temperature $T_{f1}$ = 40°C, the wall transfer coefficient being $h_{f1}$. The wall 3 is in contact with a medium at a temperature $T_{f3}$ = 35°C, the wall transfer coefficient being $h_{f3}$.
Assume steady state condition.

Data:
$a = 90$ ; $b = 4{,}500$ ; $c = -11{,}000$
thickness of the walls:
$d_1 = 0.05$ m ; $d_2 = 0.25$ m ; $d_3 = 0.15$ m
heat conductivity of the materials of the walls:
$k_1 = 10$ Wm$^{-1}$°C$^{-1}$ ; $k_2 = 15$ Wm$^{-1}$°C$^{-1}$ ; $k_3 = 30$ Wm$^{-1}$°C$^{-1}$




Calculate the temperature at the surfaces of wall 2. Where is the maximum temperature ?
Find the temperature profiles in walls 1 and 3
Calculate the values of the coefficients $h_{f1}$ and $h_{f3}$.

<u>Solution</u>
$T_2(x) = a + bx + cx^2$
$T_2(x) = 90 + 4{,}500x - 11{,}000\,x^2$

At the contact surface with wall 1 ; i.e. for $x = 0.1$ m ;
$T_2(x = 0.1) = 90 + 4{,}500\ 0.1 - 11{,}000\ 0.1^2$
$T_2(x = 0.1) = 430$°C
At the contact surface with wall 3 ; i.e. for $x = 0.35$ m ;

$T_2(x = 0.35) = 90 + 4{,}500\ 0.35 - 11{,}000\ 0.35^2$

$T_2(x = 0.35) = 317.5°C$

Temperature in both walls 1 and 3 will decrease from these values

The maximum temperature is in the wall 2, precisely:

$$\frac{dT_2(x)}{dx} = 0 = b + 2cx \Rightarrow x = -\frac{b}{2c}$$

$$x = -\frac{4,500}{2 \times 11,000} = 0.2045m$$

Temperature is there:

$T_2(x = 0.2045) = 90 + 4{,}500\ 0.2045 - 11{,}000\ 0.2045^2$

$T_2(x = 0.2045) = 550.23°C$

In the wall I temperature decreases from $T_2(x = 0.1) = 430°C$ to temperature $T_1(x = 0)$.

Continuity of heat flux at $x = 0.1$ m gives:

$$-k_1 \left.\frac{dT_1(x)}{dx}\right|_{x=0.1} = -k_2 \left.\frac{dT_2(x)}{dx}\right|_{x=0.1}$$

$$\left.\frac{dT_2(x)}{dx}\right|_{x=0.1} = 4,500 - 2 \times 11,000 \times 0.1 = 2,300°Cm^{-1}$$

$$\left.\frac{dT_1(x)}{dx}\right|_{x=0.1} = \frac{k_2}{k_1} 2,300 = \frac{15}{10} 2,300 = 3,450°Cm^{-1}$$

Temperature is linear in wall 1. So:

$$T_1(x) = a + bx$$

$$b = 3,450°Cm^{-1}$$

$$T_1(x = 0.1) = T_2(x = 0.1) = 430°C$$

$$T_1(0) = a = 430 - 3,450 \times 0.1 = 85°C$$

Heat flux through wall 1 is:

$$j_1 = k_1 \left.\frac{dT_1(x)}{dx}\right|_{x=0.1} = k_1 \left.\frac{dT_1(x)}{dx}\right|_{x=0} = 10 \times 3,450 = 34.5kWm^{-1}$$

And the heat flux exchanged with the surrounding fluid is the same:

$$j_1 = h_{f1}\left(T_1(x=0) - T_{f1}\right)$$

$$h_{f1} = \frac{j_1}{\left(T_1(x=0) - T_{f1}\right)} = \frac{34.5kWm^{-1}}{85 - 40} = 767Wm^{-1}°C^{-1}$$

Similarly, on the wall 3:

$T_2(x = 0.35) = 317.5°C = T_3(x = 0.35)$

$$- k_3 \left.\frac{dT_3(x)}{dx}\right|_{x=0.35} = - k_2 \left.\frac{dT_2(x)}{dx}\right|_{x=0.35}$$

$$\left.\frac{dT_2(x)}{dx}\right|_{x=0.35} = 4,500 - 2 \times 11,000 \times 0.35 = - 3,200°Cm^{-1}$$

$$\left.\frac{dT_3(x)}{dx}\right|_{x=0.35} = - \frac{k_2}{k_3} 3,200 = - \frac{15}{30} 3,200 = - 1,600°Cm^{-1}$$

$$T_3(x) = g + cx$$

$$c = - 1,600°Cm^{-1}$$

$$T_2(x = 0.35) = T_3(x = 0.35) = 317.5°C$$

$$g + cx = g - 1,600 \times 0.35 = 317.5$$

$$g = 1,600 \times 0.35 + 317.5 = 877.5°C$$

$$T_3(0.5) = g + c0.5 = 877.5 - 1,600 \times 0.5 = 77.5°C$$

$$j_3 = - k_3 \left.\frac{dT_3(x)}{dx}\right|_{x=0.35} = - k_3 \left.\frac{dT_3(x)}{dx}\right|_{x=0.5} = 30 \times 1,600 = 48kWm^{-1}$$

$$j_3 = h_{f3}\left(T_3(x = 0.5) - T_{f3}\right)$$

$$h_{f3} = \frac{j_3}{\left(T_3(x = 0.5) - T_{f3}\right)} = \frac{48kWm^{-1}}{77.5 - 35} = 1129Wm^{-1}°C^{-1}$$

**Mechanics of solids & Materials**

Forces
Moments
Couples
Static equillibrium
Free body method of analysis
Friction, internal forces, stress and strain, Hooke's law
Kinematics and kinetics of particles and rigid bodies
Moving frames of reference
Newton's laws
Conservation of energy and momentum