Signal and Systems

This problem involves a quadri-rotor operating in a quasi-stationary way, as in figure 1. It is divide into 2 parts that are independent.

Figure 1. Quadri-rotor

To simplify, consider that the body only moves in a vertical plane. As a result, the problem can be studied as if there were only two rotors, as in figure 2.

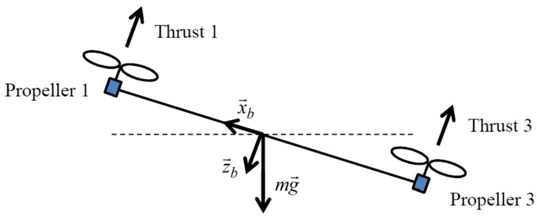

Figure 2. Simplified model

Each propeller is driven by a DC motor and supplies a thrust proportional to the square of its rotation speed. To make the drone ascend or move down, one has to act on the sum of the thrusts; to go left or right, one has to act on the difference of the thrusts. Further, a propeller creates another force (called drag of the rotor) in the body plane; this force is opposed to the linear speed of the drone and is proportional to the product (linear speed×rotation speed). The other aerodynamical effects can be neglected when the drone is operated at low speeds.
The Fundamental Principle of Dynamics, in the frame $(\vec{x}_b, \overrightarrow{z_b})$ of the drone, states that:

$$m(\dot{u} + w\dot{\theta}) = -mg\, sin(\theta) - \mu_1(\Omega_1 + \Omega_3)u$$
$$m(\dot{w} - u\dot{\theta}) = mg\, cos(\theta) - a(\Omega_1^2 + \Omega_3^2)$$
$$J\ddot{\theta} = al(\Omega_1^2 - \Omega_3^2) + \mu_4(\Omega_1 + \Omega_3)u - \mu_2(\Omega_1 + \Omega_3)\dot{\theta}$$

where u and w are the components of the speed of the inertia center in the body frame, θ is the angle of the drone with respect to the horizontal axis, Ω_1 et Ω_3 are the rotation speeds of the propellers (Ω_1 and Ω_3 have to be positive). Consider that Ω_1 and Ω_3 are the inputs of the system. Further, u and w are measured.

Part 1:

Question 1.1
Give a state space representation of the system. One can note $\dot{\theta} = q$.

Question 1.2
Calculate the values of the state variables and the inputs which correspond to the stationary flight equilibrium point, that is $\bar{u} = \bar{w} = 0$.

Question 1.3
Linearize the system about this equilibrium point. Variations about equilibrium point will be noted using the symbol δ (that is, if the variable $z(t)$ is equal to $\bar{z}$ for the equilibrium, the notation will be $z(t) = \bar{z} + \delta z(t)$.

Question 1.4
Using the following change of variables:

$$\delta\Omega^+ = \delta\Omega_1 + \delta\Omega_3$$
$$\delta\Omega^- = \delta\Omega_1 - \delta\Omega_3$$

Prove that the system can be decomposed into two decoupled systems, namely « vertical move » and « horizontal move » and give the new equations.

Part 2 :

Considering specific numerical values of parameters for the quadri-rotor, leads to the following equation for the horizontal move subsystem :

$$\begin{bmatrix} \delta\dot{u} \\ \delta\dot{\theta} \\ \delta\dot{q} \end{bmatrix} = \begin{bmatrix} -5\times10^{-4}\bar{\Omega} & -10 & 0 \\ 0 & 0 & 1 \\ 2\times10^{-3}\bar{\Omega} & 0 & -20\bar{\Omega} \end{bmatrix} \begin{bmatrix} \delta u \\ \delta\theta \\ \delta q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 200/\bar{\Omega} \end{bmatrix} \delta\Omega^-$$

$$\delta y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta u \\ \delta\theta \\ \delta q \end{bmatrix}$$

Where $\bar{\Omega} = \sqrt{\frac{mg}{2a}}$ corresponds to the rotational speed of the propellers for the stationary flight. a is a dimensioning parameter which should be chosen by the designer.

Question 2.1
Study the stability of the equilibrium point with respect to the value of the speed $\bar{\Omega}$. Is this condition hard to satisfy?

Question 2.2
Calculate the transfer function of the system $\frac{\delta u(s)}{\delta\bar{\Omega}(s)}$

Note : it should be noted that only the expression of a_{13} is required to multiply these matrices:

$$\begin{bmatrix} c & 0 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}$$

Question 2.3
Using the following property:

$$s^3 + 20.0005\,\bar{\Omega}\,s^2 + 10^{-2}\,s + 0.02\bar{\Omega} = (s + 20\bar{\Omega})(s^2 + 0.0005\bar{\Omega}s + 0.001)$$

Prove that it is possible to simplify the expression of the transfer function in the case of realistic conditions, which should be given.

Question 2.4
It is decided that $\Omega = 50 rad.s^{-1}$. Give the Bode diagram of the system together with its step response.
In particular, the time response of the system should be given. (Note: 0.001 ≈ 0.03).

Question 2.5
It is considered that acceptable behavior of such a system corresponds to a time response of 10 seconds and an overshoot of less than 5%. Choosing a different value for Ω , is it possible to achieve such performances?