



Electrical Engineering

2 hours

The use of a calculator is allowed

The test is composed of **3** parts

Part 1 Electronics

Exercise 1:

Digital electronics: *Sequence generator design*

The purpose is to generate a sequence of bits *SEQ* **synchronous** with a clock signal *Clk*, consisting of a pattern of 8 bits that is repeated indefinitely.

The pattern sequence is: 0 0 0 1 1 1 0 1, so that the signal *SEQ* should be
0 0 0 1 1 1 0 1 0 0 0 1 1 1 0 1 0 0 0 1 1 1 0 1.....

This signal should be generated from a synchronous finite-state machine, composed of D flip-flops, inverters, and classic gates (AND, OR, NAND, NOR).

The signal *SEQ* should be directly the output of a D flip-flop, and the number of D flip-flops should be **minimized**.

- 1.1 Draw the state diagram of the state machine and detail the coding of each state.
- 1.2 How many states and D flip-flops are needed to make such a generator?
- 1.3 Synthesize the machine. Rising-edge triggered D flip-flops will be used for the state variables.
- 1.4 Determine the maximal clock frequency that guarantees a correct operation of the generator.
The timing characteristics of the gates and flip-flops are given in Table I.

	tpLH (ns)			tpHL (ns)			tset-up (ns)	thold (ns)
	min	typ	max	min	typ	max		
D-type Flip-Flop D to Clock Pulse CP to Q or \Q	-	12	20	-	18	30	20	6
n-inputs Gate (AND, OR, NAND, NOR)	-	15	20	-	15	20		
Inverter	-	5	8	-	7	10		

Table I

Exercise 2:

Analogue electronics : *Sallen-key filter*

Figure 1 illustrates a Sallen-Key low-pass filter schematic. Assume that the operational amplifier is ideal with input resistance R_{in} very large and output resistance R_{out} negligibly small, so that $i_{in}^+ \approx i_{in}^- \approx 0$, and $V_{out} = A_V(v_{in}^+ - v_{in}^-)$, with A_V very large. Assume it is operating in its linear range.

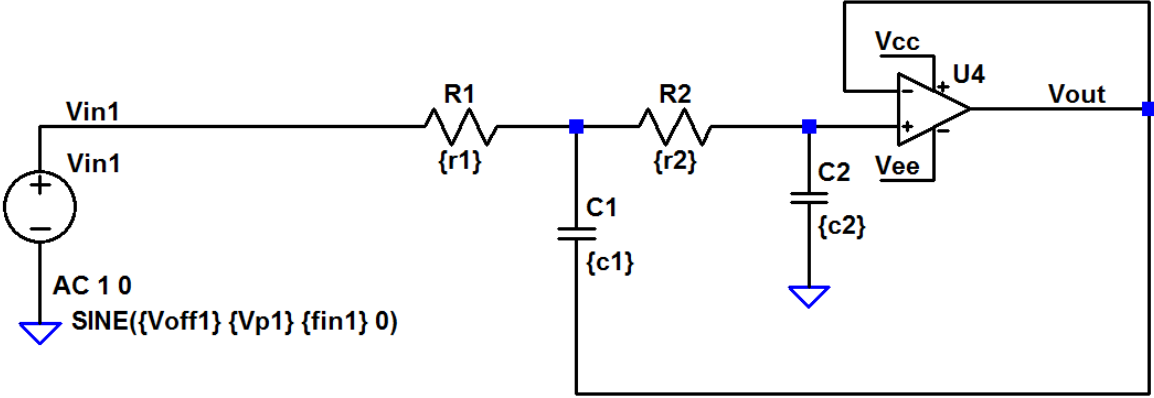


Fig. 1. Sallen-Key biquad filter schematics

- 2.1 Derive the analytical expression of the filter transfer function in terms of R_1, R_2, C_1, C_2 by supposing that A_v is infinite.
- 2.2 Propose a circuit sizing for resistors and capacitors to have a cut-off frequency at 1 kHz. Use commercial values for R_1, R_2, C_1, C_2 .
- 2.3 Operational amplifiers are implemented using advanced MOS transistors. Draw a cross-section view of a NMOS transistor and roughly explain how it works in the linear region and in the saturation region.
- 2.4 Propose a simple transistor-level implementation for the operational amplifier. Explain very simply the role of each transistor that appears in the schematic.

Part 2 Dynamical System Modelling and Analysis

Exercise 1: (questions are independent of each other)

1) We consider the system modelled by the transfer function $H(s) = \frac{1}{1+\tau s}$.

What is the condition on τ that ensures that the system is stable?

2) We consider the system modelled by the state space equations

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

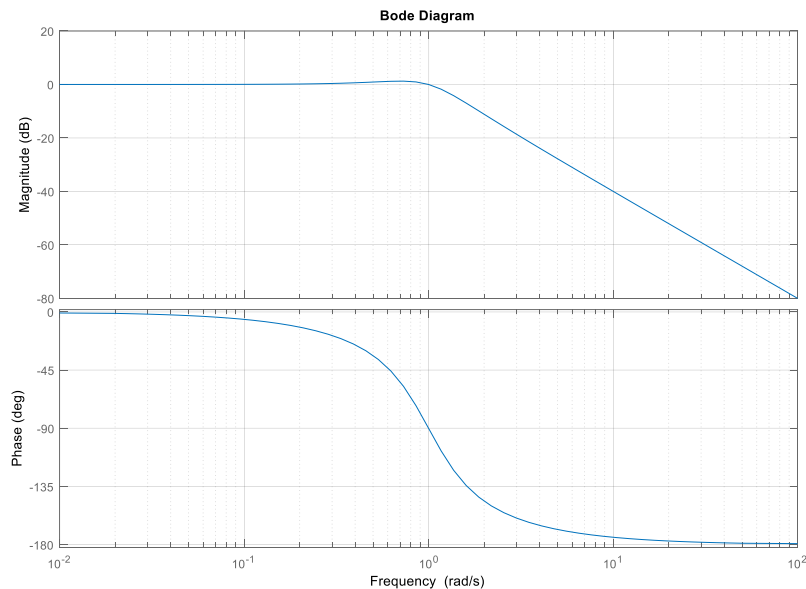
$$y = (2 \quad 1)x$$

give the transfer function $\frac{y(s)}{u(s)}$

3) We consider the system modelled by the transfer function $H(s) = \frac{-1}{1-s+s^2}$

Give the final value of the step response of this system.

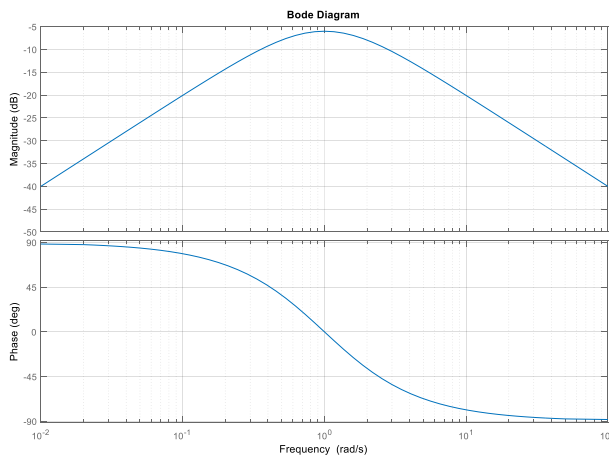
4) The Bode plot of the transfer function $H_1(s) = \frac{1}{1+s+s^2}$ is given:



Give the values of k and ω_0 such that for the transfer function $H_2(s) = \frac{k \cdot \omega_0^2}{s^2 + \omega_0 s + \omega_0^2}$:

- The phase is equal to -90° at $\omega = 10 \text{ rad/s}$
- The gain is equal to 10 at $\omega = 0.1 \text{ rad/s}$

5) We consider a linear system the bode plot of which is given:



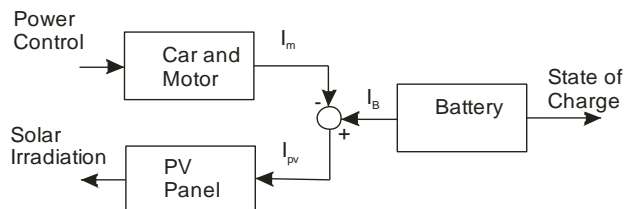
The input signal is $u(t) = 2\sin t + \sin(0.1t)$. What is the output signal $y(t)$?

Exercise 2:

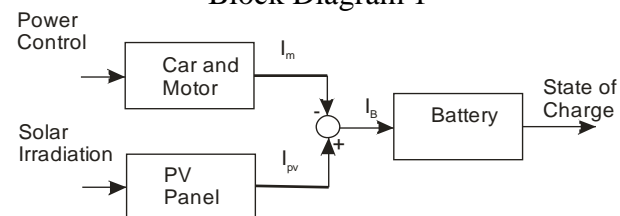
We consider the electrical system of a solar car composed by:

- A PV generation unit: the current generated by this unit is considered as proportional to the solar irradiation.
- A battery that can be charged or discharged.
- An electrical motor used to move the car. This motor is controlled by the electrical power it consumes.
- All these components are connected to a DC bus with constant voltage.

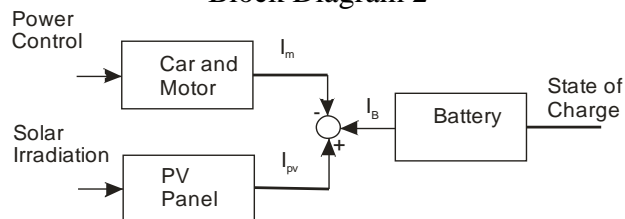
1) 3 Block Diagram models are proposed to represent this system. Which one is the good one? Why?



Block Diagram 1

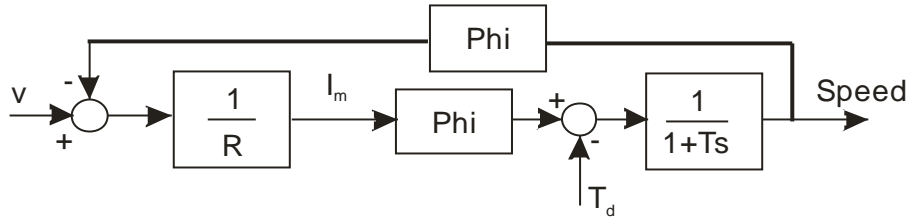


Block Diagram 2



Block Diagram 3

- 2) The battery is modelled as an integrator $\frac{SOC}{I_B} = \frac{k_b}{s}$ where SOC denotes the state of charge and the PV panel as a gain K . Give the transfer function $\frac{SOC}{SI}$ where SI denotes the solar irradiation.
- 3) The motor and the car are modelled by the block diagram below



Give the transfer functions $\frac{I_m}{V}$ and $\frac{I_m}{T_d}$

- 4) Explain whether this model is consistent with the model of the global system.

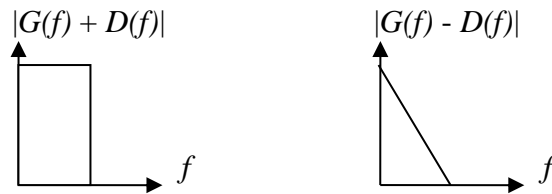
Part 3 Signals

Let's consider two real signals $g(t)$ and $d(t)$ having their spectrum limited to $[-15kHz, +15kHz]$.

The signal $s(t)$ is defined by

$$s(t) = [g(t) + d(t)] + [g(t) - d(t)]\cos(2\pi f_s t) \text{ with } f_s = 38kHz.$$

1. We denote $G(f)$ and $D(f)$ the Fourier Transform of $g(t)$ and $d(t)$ and we consider that for the positive frequencies we have the following spectrums:



-What is the representation of those spectrums for the negative frequencies?

-Give the expression of $S(f)$, the Fourier Transform of $s(t)$, as a function of $G(f)$ and $D(f)$ and draw $|S(f)|$.

-What is the frequency occupation of $s(t)$.

2. If we sample $s(t)$ at the frequency f_s , do we respect the Nyquist rate?

-Let's obtain two signals by doing the two following samplings:

$$s_1[k] = s(k.T_s)$$

$$s_2[k] = s\left(k.T_s + \frac{T_s}{2}\right)$$

where $T_s = 1/f_s$.

-by using the expression in the time domain show that those samplings allow to separate the signals g and d .

-demonstrate it also by explaining what happens in the frequency domain.