



Mathematics and Physics MCQ
Mathematics & Computer Science Track

Duration: 1 hour

Scoring scale: 2 for a correct answer, 0 if no answer is given, -1 for a wrong answer

For some questions, you have to fill several good answers.

A standard -non scientific- language dictionary is authorized. Please make sure to have it checked by the staff. Documents, electronic devices and calculators are not allowed.

Notations

\mathbb{R} is the set of real numbers, \mathbb{C} is the set of complex numbers. For $a, b \in \mathbb{R}$,

- $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$, $(a, b) = \{x \in \mathbb{R} : a < x < b\}$,
- $[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$, $(a, b] = \{x \in \mathbb{R} : a < x \leq b\}$,
- $\mathcal{M}_n(\mathbb{K})$ is the set of square matrices of order n on the field \mathbb{K} .

Question 1 The sequence $(\sum_{k=1}^n (1/k^{3/2}))_{n \in \mathbb{N}^*}$ has a finite limit.

- A False
 B True

Question 2 Any union of closed subsets of \mathbb{R} is a closed set.

- A False B True

Question 3 Let $(u_n)_{n \in \mathbb{N}}$ be a real sequence such that the subsequences $(u_{6n+2})_{n \in \mathbb{N}}$ and $(u_{3n+1})_{n \in \mathbb{N}}$ converge. Then the sequence $(u_n)_{n \in \mathbb{N}}$ converges.

- A True B False

Question 4 Consider the sequence of functions $f_n : x \mapsto (1-x)^n$ on the segment $[0, 1]$, $n \geq 0$. Then

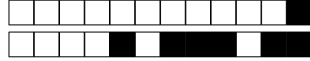
- A $(f_n)_{n \geq 0}$ converges uniformly but not pointwise
 B $(f_n)_{n \geq 0}$ converges pointwise and uniformly
 C $(f_n)_{n \geq 0}$ converges pointwise but not uniformly

Question 5 The radius of convergence of the power series $\sum n^4 x^n$ is equal to

- A 1/4 C $+\infty$
 B 4 D 1

Question 6 What is the radius of convergence of the power series $\sum_{n \geq 1} (1+7^n)x^n/n^2$?

- A 1 C 0
 B $+\infty$ D $\frac{1}{7}$



Question 7 The Taylor expansion at 0 of any infinitely differentiable function on \mathbb{R} has a positive radius of convergence.

- A False B True

Question 8 Let A be a real matrix of order $n \geq 2$ such that $A^2 + A + I_n = 0$, where I_n is the identity matrix of order n . Then A is diagonalizable in \mathbb{C} .

- A True B False

Question 9 For all $n \in \mathbb{N}$, we consider the sequence of functions $(u_n)_{n \in \mathbb{N}}$ defined by

$$\forall x \in \mathbb{R}, \quad u_n(x) = n x^n.$$

Which of these statements are true?

- A The sum $\sum_{n=1}^{\infty} u_n$ is continuous at $x = 1$
 B The series $\sum \int_{-1}^1 u_n(x) dx$ converges
 C The series $\sum u_n$ converges normally on \mathbb{R}
 D The series $\sum u_n$ converges normally on $[-r, r]$, for all $0 < r < 1$

Question 10 Let A be a positive definite symmetric matrix of order $n \times n$ and let D be a diagonal matrix, whose diagonal elements $(D_{i,i})_{1 \leq i \leq n}$ satisfy $D_{i,i} \geq 0$ for all $1 \leq i \leq n$.

The product AD is diagonalizable and its eigenvalues are real and positive.

- A True B False

Question 11 Let $\mathcal{B} = (e_1, \dots, e_{10})$ and $\mathcal{B}' = (e'_1, \dots, e'_{10})$ be two distinct bases of \mathbb{R}^{10} .

Let $f : \mathbb{R}^{10} \rightarrow \mathbb{R}^{10}$ be an endomorphism such that the matrix de f related to the bases \mathcal{B} and \mathcal{B}' is the identity matrix I_{10} . Is the endomorphism f the identity of \mathbb{R}^{10} ?

- A Yes C No
 B Unable to conclude

Question 12 Let E and F be two vector spaces of dimensions n and p respectively.

Let $\mathcal{L}(E, F)$ denote the vector space of linear mappings $E \rightarrow F$, $\mathcal{L}(E \times F)$ the vector space of endomorphisms of $E \times F$, $\mathcal{L}(E)$ the vector space of endomorphisms of E et $\mathcal{L}(F)$ the vector space of endomorphisms of F .

Which of these statements are true?

- A $\mathcal{L}(E \times F)$ has dimension $n + p$
 B $\mathcal{L}(E \times F)$ is isomorphic to $\mathcal{L}(E) \times \mathcal{L}(F)$
 C $\mathcal{L}(E \times F)$ has dimension $(n + p)^2$
 D $\mathcal{L}(E, F)$ is isomorphic to $\mathcal{L}(E) \times \mathcal{L}(F)$
 E $\mathcal{L}(E \times F)$ has dimension np



Question 13 Let A be a square real matrix of order 3×3 , for which 1 and -1 are eigenvalues. We assume the eigenspace related to the eigenvalue 3 has dimension 1 , and the eigenspace related to the eigenvalue -1 has dimension 1 .

Which of these statements are true?

- A If 1 and 3 are not the only eigenvalues of A , then A is diagonalizable
- B If 1 and 3 are not the only eigenvalues of A then A is invertible
- C If 1 and 3 are the only eigenvalues of A then A is similar to a triangular matrix
- D The characteristic polynomial of A is the product of polynomials of degree 1
- E If 1 and 3 are the only eigenvalues of A then A is diagonalizable

Question 14 Let A be a matrix in $\mathcal{M}_3(K)$, where either $K = \mathbb{R}$ or $K = \mathbb{C}$ and $A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$.

Which of these statements are true?

- A If $K = \mathbb{C}$ then A is similar to a triangular matrix but A is not diagonalizable
- B If $K = \mathbb{R}$ then A is not diagonalizable
- C If $K = \mathbb{R}$ then A is similar to a triangular matrix
- D The characteristic polynomial of A is the product of polynomials of order 1 if $K = \mathbb{C}$ and if $K = \mathbb{R}$
- E If $K = \mathbb{C}$ then A has 3 different eigenvalues

Question 15 Let $\mathcal{F} : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a function defined by $\mathcal{F}(x, y) = (2x + y, x - y, |x| - y)$. Is \mathcal{F} linear?

- A No
- B Yes

Question 16 Let F be a vector subspace of a vector space E . Then, $\forall (x, y) \notin F^2, x + y \notin F$.

- A False
- B True

Question 17 The inverse of a lower triangular invertible matrix is upper triangular.

- A False
- B True

Question 18 Let $u \in C^4([0, 1])$, $h \in (0, 1)$ and $x \in (h, 1 - h)$. Then $|h^2 u''(x) - (u(x + h) - 2u(x) + u(x - h))| \leq \frac{h^4}{12} \max_{x \in [0, 1]} |u^{(4)}(x)|$.

- A True
- B False

Question 19 Let f be an endomorphism of \mathbb{R}^2 such that $f((0, 1)) = (-1, 1)$ and $f((-1, 1)) = (-1, 0)$. The matrix of f in the basis $((0, 1), (-1, 1))$ is :

- A $\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$
- B $\begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$
- C $\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$

Question 20 In the most general case the sum of the forces applied to a system is equal to:

- A $\frac{d\vec{p}}{dt}$ with \vec{p} its momentum.
- B $m \cdot \vec{a}$, with m its mass and \vec{a} its acceleration.
- C $\frac{1}{2} m \cdot v^2$, with m its mass and v its speed.
- D $\frac{d\vec{p}}{dt}$ with \vec{p} its momentum.



Question 21 A vacationer jumps from a bridge at height h above a river. His speed when he reaches the water is:

- A $2.g.h$. C $\sqrt{2.g.h}$.
 B $\sqrt{2.m.g}$. D $2.m.g$.

Question 22 During a car crash, if the car has a speed two times larger than the speed restriction, the shock is:

- A four times more violent. C two times more violent.
 B ten times more violent. D the same.

Question 23 A potential energy of expression $E_p = \frac{\alpha}{\sqrt{2}}x + \beta$ is associated to a force:

- A $-\frac{\alpha}{2^{3/2}}$. C $\alpha\sqrt{2}$.
 B $\frac{\alpha}{2^{3/2}}$. D $-\alpha\sqrt{2}$.

Question 24 Planets have elliptical orbits. The conservation of angular momentum imposes that:

- A their speed is higher when they reach the extremities of the major axis of their orbit. C all the planets have the same rotation speed.
 B their speed is higher when they reach the extremities of the minor axis of their orbit. D their rotation speed is the same all along the orbit.

Question 25 A moment of inertia has the units of

- A A frequency divided by a momentum C all the planets have the same rotation speed.
 B a surface divided by an energy D energy divided by the square of a frequency

Question 26 When an electron travels between two electrodes, the left one being at a higher electric potential than the right one:

- A it moves toward the right. C it moves downward.
 B it moves upward. D it moves toward the left.

Question 27 A diatomic molecule is described as an harmonic oscillator in the air ($\omega \sim 10^{15}$ Hz), the typical displacement speed of the atoms is 3×10^5 m.s⁻¹. The typical vibration amplitude is:

- A $A_{\max} = 300 \mu\text{m}$. C $A_{\max} = 300 \text{nm}$.
 B $A_{\max} = 300 \text{mm}$. D $A_{\max} = 300 \text{pm}$.

Question 28 The angular momentum of the gravity force of a pendulum with length l and mass m forming an angle α with the vertical axis is:

- A $mgl \cos \alpha$. C $\frac{gl \sin \alpha}{m}$.
 B $mgl \sin \alpha$. D $\frac{gl \cos \alpha}{m}$.

Question 29 If g is the gravity acceleration, the oscillation period of a pendulum of length l and mass m is:

- A $\sqrt{\frac{g}{m}}$. C $\sqrt{\frac{l}{m}}$.
 B $\sqrt{\frac{l}{g}}$. D $\sqrt{\frac{g}{l}}$.



Question 30 A body has a displacement which is proportional to the time elapsed. The force acting on it is:

- A Opposite to the speed vector. D In the same direction as the displacement
 B Nil E In the same direction as the speed vector
 C Opposite to the displacement

Question 31 A 20 g bullet hits an initially immobile target made of lead. The lead block weighs 10 kg and is hanging from the ceiling attached by a string. The bullet is trapped after it has penetrated inside the block. After it has caught the bullet, the block oscillates so that it raises at maximum 4 cm above its equilibrium position. What was the initial speed of the bullet?

- A 200 m/s D 10 m/s
 B 100 m/s E 500 m/s
 C 450 m/s

Question 32 A piece of copper weights 2 kg. It is dropped into a bucket of water of 2 litres from a height of 3 meters. Assuming that both systems were initially at room temperature ($T = 20\text{ C}$), what is the energy transferred to the global system once the equilibrium has been reached? .

- A 0 Joule because it is now equilibrium C 0 joule because because the temperature has not changed.
 B 30 Joules D 60 Joules

Question 33 A marble is falling from a height of 13 meters. Because the marble is not infinitely rigid nor elastic, it bounces back up only to the third of the initial height. And bounces again from the third of the new height. And so on. The specific heat of the marble is 0.13 kJ/kg.K . What is the increase of its temperature after the second bounce?

- A 50 C D 10 C
 B 0.9 C E 0.2 C
 C 2 C

Question 34 The pressure of a gas in terms of its mean kinetic energy per unit volume U is equal to

- A $\frac{4U}{5}$ E $\frac{5U}{4}$
 B $\frac{U}{3}$ F $\frac{2U}{3}$
 C $\frac{U}{2}$ G $\frac{4U}{3}$
 D $\frac{3U}{2}$

Question 35 Let R be the the ideal gas constant $R \approx 8.3\text{ J/mol.K}$. To increase the temperature of two moles of a perfect mono-atomic gas by 1 degree at constant volume, it is necessary to bring a heat of

- A $5R$ D $2R$
 B $\frac{7}{3}R$ E $3R$
 C $\frac{5}{2}R$

Question 36 Which of the following is NOT a property of the system

- A specific volume D density
 B pressure E heat
 C temperature



Question 37 Work done in an adiabatic process between a given pair of end states depends on

- A the end states only
- B the value of heat transferred
- C the type of forces
- D particular adiabatic process
- E the kinetics of the process

Question 38 A perfect gas at 27 C is heated at constant pressure till its volume is doubled. The final temperature is

- A 327 C
- B 54 C
- C 600 C
- D 654 C
- E 108 C



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← Encode your inscription number on the left, and write your name and first name in the field below.

Name and first name:
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Answers must be given exclusively on this sheet: answers given on the other sheets will be ignored.

WARNING: the boxes must be completely filled, not checked, otherwise your answers might not be taken into account.

EXAMPLE: **Fill the box :** Question 16: A B **OK**

Question 23: A B **NO!**

ANSWERS:

QUESTION 1 : A B

QUESTION 2 : A B

QUESTION 3 : A B

QUESTION 4 : A B C

QUESTION 5 : A B C D

QUESTION 6 : A B C D

QUESTION 7 : A B

QUESTION 8 : A B

QUESTION 9 : A B C D

QUESTION 10 : A B

QUESTION 11 : A B C

QUESTION 12 : A B C D E

QUESTION 13 : A B C D E

QUESTION 14 : A B C D E

QUESTION 15 : A B

QUESTION 16 : A B

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QUESTION 36 : A B C D E

QUESTION 37 : A B C D E

QUESTION 38 : A B C D E



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