CentraleSupélec
Accelerated Engineering Degree Programme
Master's level

September 2018

Mathematics MCQ

Physics, Mechanical Engineering & Electrical Engineering Tracks

Duration: 1 hour

Scoring scale: 2 for a correct answer, 0 if no answer is given, -1 for a wrong answer

For some questions, you have to fill several good answers.

A standard -non scientific- language dictionary is authorized. Please make sure to have it checked by the staff. Documents, electronic devices and calculators are not allowed.

Notations

$\mathbb{R}$ is the set of real numbers, $\mathbb{C}$ is the set of complex numbers. For $a, b \in \mathbb{R}$,

- $[a,b] = \{x \in \mathbb{R} : a \leq x \leq b\}$, $(a,b) = \{x \in \mathbb{R} : a < x < b\}$,
- $[a,b) = \{x \in \mathbb{R} : a \leq x < b\}$, $(a,b] = \{x \in \mathbb{R} : a < x \leq b\}$,
- $\mathcal{M}_n(\mathrm{K})$ is the set of square matrices of order n on the field K.

Question 1 Let X, Y be two sets and let $f : \mathrm{X} \to \mathrm{Y}$ be a function.
Then, for all subsets B_1 and B_2 of Y, $f^{-1}(\mathrm{B}_1 \cup \mathrm{B}_2) = f^{-1}(\mathrm{B}_1) \cup f^{-1}(\mathrm{B}_2)$.

A False   B True

Question 2 Let E, F be 2 $\mathbb{R}$-vector spaces of finite dimension, $\dim(\mathrm{E}) = n$, $\dim(\mathrm{F}) = p$. Let f be a linear map from E to F. Then

A $\mathrm{rank}(f) = n + \dim(\mathrm{Ker}(f))$

B $p = \dim(\mathrm{Ker}(f)) + \mathrm{rank}(f)$

C $\dim(\mathrm{Ker}(f)) = p + \mathrm{rank}(f)$

D $n = \dim(\mathrm{Ker}(f)) + \mathrm{rank}(f)$

Question 3 Any square real matrix which is symmetric is diagonalizable and has only real eigenvalues.

A True   B False

Question 4 The matrix $\mathrm{J} = (1)_{1 \leq i,j \leq n}$, $n \geq 2$

A is diagonalizable

B counts 1 in its eigenvalues

C of rank 1

D is invertible

Question 5 Does the series $\sum_{n \geq 2} \frac{1}{n(\ln n)^2}$ converge?

A No   B Yes

Question 6 Let $f : x \in \mathbb{R} \mapsto \log\left(\dfrac{1}{\exp(-x)+1}\right)$. Then

- [A] $\lim_{x\to+\infty} f(x) = 0$
- [B] $\lim_{x\to+\infty} f(x)$ does not exist
- [C] $\lim_{x\to+\infty} f(x) = -\infty$
- [D] $\lim_{x\to+\infty} f(x) = +\infty$

Question 7 $\mathbb{Q}$ is countable.

- [A] False
- [B] True

Question 8 $\mathbb{R}$ is countable.

- [A] False
- [B] True

Question 9 Recall that $\subsetneq$ denotes: "subset of and not equal to". Let E be a set, A a subset of E and $(\mathrm{B}_i)_{i\in\mathrm{I}}$ a collection of subsets of E, indexed by an unspecified set I. Then

- [A] $\bigcup_{i\in\mathrm{I}}(\mathrm{A}\cap\mathrm{B}_i) \subsetneq \mathrm{A}\cap(\bigcup_{i\in\mathrm{I}}\mathrm{B}_i)$
- [B] $\mathrm{A}\cap(\bigcup_{i\in\mathrm{I}}\mathrm{B}_i) \subsetneq \bigcup_{i\in\mathrm{I}}(\mathrm{A}\cap\mathrm{B}_i)$
- [C] $\mathrm{A}\cap(\bigcup_{i\in\mathrm{I}}\mathrm{B}_i) = \bigcup_{i\in\mathrm{I}}(\mathrm{A}\cap\mathrm{B}_i)$

Question 10 For $(x,y)\in\mathbb{R}^2$, we define $f(x,y) = (\sin(x)+\sin(y), (x+y)/(x^2+y^2))$. Then f is

- [A] bounded on $\mathbb{R}^2$
- [B] continuous over $\mathbb{R}^2$
- [C] symmetric

Question 11 Let $(u_n)_{n\in\mathbb{N}}$ and $(v_n)_{n\in\mathbb{N}}$ be two real sequences which converge to l_1 and l_2 respectively. If for all $n\in\mathbb{N}$, $u_n < v_n$ then

- [A] $l_1 < l_2$
- [B] $l_1 = l_2$
- [C] $l_1 \leq l_2$

Question 12 Let X be a random variable with nonnegative integer values. It is said that X follows a geometric law if

- [A] there exists a real number $\lambda > 0$ such that $\forall k\in\mathbb{N}, \mathbb{P}(\mathrm{X}=k) = e^{-\lambda}\dfrac{\lambda^k}{k!}$.
- [B] there exists a real number $p\in(0,1)$ such that $\forall k\in\mathbb{N}, \mathbb{P}(\mathrm{X}=k) = (1-p)^k p$.
- [C] there exists a real number $p\in(0,1)$ such that $\forall k\in\mathbb{N}, \mathbb{P}(\mathrm{X}=k) = \binom{n}{k} p^k(1-p)^{n-k}$.

Question 13 The set $\bigcup_{n=1}^{\infty}\left[\dfrac{1}{n},1\right]$ is equal to

- [A] $(0,1]$
- [B] $[0,1]$

Question 14 Let A be a matrix in $\mathscr{M}_5(\mathbb{R})$. For all $\mathrm{B},\mathrm{C}\in\mathscr{M}_5(\mathbb{R})$, we have $(\mathrm{AB}=\mathrm{AC}) \Rightarrow (\mathrm{B}=\mathrm{C})$.

- [A] False
- [B] True

Question 15 Let $(u_n)_{n\in\mathbb{N}}$ be a real sequence. Which of the following statements are true ?

- [A] If $\dfrac{u_n}{1+u_n^2} \to 0$ then u_n converges to 0, as $n \to \infty$
- [B] If $\lim\limits_{n\to\infty} u_n = \ell$ then $u_{n+1} - u_n$ converges to 0, as $n \to \infty$
- [C] The sequence $(v_n)_{n\in\mathbb{N}}$ defined by $v_n = \dfrac{u_n}{1+u_n^4}$ is bounded
- [D] If $u_{n+1} - u_n$ converges to 0 then $(u_n)_{n\in\mathbb{N}}$ converges
- [E] If $\dfrac{u_{n+1}}{u_n} \to 1$ then $(u_n)_{n\in\mathbb{N}}$ converges

Question 16 Let $f : x \in (-1,1) \mapsto \dfrac{e^x - 1}{(x+1)^2 - 1}$. Then

- [A] $\lim\limits_{x\to 0} f(x) = 1/2$
- [C] $\lim\limits_{x\to 0} f(x) = +\infty$
- [B] $\lim\limits_{x\to 0} f(x) = 0$
- [D] $\lim\limits_{x\to 0} f(x) = 2$

Question 17 For all $x \in [0, \pi/2]$, $\sin(x) \geq 2x/\pi$.

- [A] False
- [B] True

Question 18 Let $F : \mathbb{R} \to \mathbb{R}^2$ be a function defined by $F(t) = G(2t+1, e^t)$ where $G : \mathbb{R}^2 \to \mathbb{R}$ is defined by $G(x,y) = x^2 + y$. What is the expression of $F'(t)$ for $t \in \mathbb{R}$?

- [A] $2(2t+1) + e^t$
- [C] $4(2t+1) + e^t$
- [B] $2 + e^t$

Question 19 Let f be a continuous function on $\mathbb{R}$ such that $\int_0^{+\infty} f(x)dx$ is convergent. Then we have necessarily $f(x) \to 0$, as $x \to +\infty$.

- [A] False
- [B] True

Question 20 The set of functions $\big(x \mapsto \sin(n\pi x)\big)_{n\geq 1}$ is an orthonormal collection for the scalar product $\langle f, g\rangle = \displaystyle\int_{-1}^{1} f(x)g(x)\mathrm{d}x$.

- [A] True
- [B] False

Question 21 The function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = \dfrac{xy}{\sin x}$ if $x \neq k\pi$ and $f(k\pi, y) = y$, for $k \in \mathbb{Z}$, is continuous at $(0,0)$.

- [A] False
- [B] True

Question 22 The problem

$$\begin{cases} -u' = 1 & \text{in} \quad]0,1[, \\ u(0) = 0 & \text{and} \quad u'(1) = 0. \end{cases}$$

has

- [A] a unique solution $x \mapsto x(2-x)/2$
- [B] no solution
- [C] several solutions

Question 23 The matrix $\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ is symmetric and positive definite.

- [A] True
- [B] False

Question 24 Let $\mathscr{B} = (e_1, e_2, e_3, e_4)$ be a basis of $\mathbb{R}^4$. Let f be the endomorphism of $\mathbb{R}^4$ whose matrix related to the basis $\mathscr{B}$ is: $\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$. Is the map f bijective?

- [A] No
- [B] Yes

Question 25 Let E and F two vector spaces. Consider two linear mappings f and g from E to F such that there exists an endomorphism h of E which satisfies $g = f \circ h$.
Which of the following inclusions are satisfied?

- [A] $\operatorname{Im} g \subset \operatorname{Im} f$
- [B] $\operatorname{Ker} f \subset \operatorname{Ker} g$
- [C] $\operatorname{Im} f \subset \operatorname{Im} g$
- [D] $\operatorname{Ker} g \subset \operatorname{Ker} f$

Question 26 Let $\mathscr{F} : \mathbb{R}^2 \to \mathbb{R}^3$ be a function defined by $\mathscr{F}(x, y) = (2x + y, x - y, |x| - y)$
Is $\mathscr{F}$ linear?

- [A] Yes
- [B] No

Question 27 Let F be s vector subspace of a vector space E. Then, $\forall (x, y) \not\in F^2, x + y \not\in F$.

- [A] True
- [B] False

Question 28 The inverse of a lower triangular invertible matrix is upper triangular.

- [A] True
- [B] False

Question 29 Let $u \in C^4([0,1])$, $h \in (0,1)$ and $x \in (h, 1-h)$. Then $|h^2 u''(x) - (u(x+h) - 2u(x) + u(x-h))| \leq \dfrac{h^4}{12} \max_{x\in[0,1]} |u^{(4)}(x)|$.

- [A] True
- [B] False

Question 30 Let f be an endomorphism of $\mathbb{R}^2$ such that $f((0,1)) = (-1,1)$ and $f((-1,1)) = (-1,0)$.
The matrix of f in the basis $\big((0,1), (-1,1)\big)$ is :

- [A] $\begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$
- [B] $\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$
- [C] $\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$

CentraleSupélec
Accelerated Engineering Degree Programme
Master's level

September 2018

Mathematics MCQ

Physics, Mechanical Engineering & Electrical Engineering Track

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

⟵ Encode your inscription number on the left, and write your name and first name in the field below.

Name and first name:
....

Answers must be given exclusively on this sheet: answers given on the other sheets will be ignored.

WARNING: the boxes must be completely filled, not checked, otherwise your answers might not be taken into account.

EXAMPLE:

Fill the box : Question 16 : A ■ **OK** Question 23 : A ▨ **NO !**

ANSWERS:

QUESTION 1 : A B
QUESTION 2 : A B C D
QUESTION 3 : A B
QUESTION 4 : A B C D
QUESTION 5 : A B
QUESTION 6 : A B C D
QUESTION 7 : A B
QUESTION 8 : A B
QUESTION 9 : A B C
QUESTION 10 : A B C
QUESTION 11 : A B C
QUESTION 12 : A B C
QUESTION 13 : A B
QUESTION 14 : A B
QUESTION 15 : A B C D E
QUESTION 16 : A B C D
QUESTION 17 : A B
QUESTION 18 : A B C
QUESTION 19 : A B
QUESTION 20 : A B
QUESTION 21 : A B
QUESTION 22 : A B C
QUESTION 23 : A B
QUESTION 24 : A B
QUESTION 25 : A B C D
QUESTION 26 : A B
QUESTION 27 : A B
QUESTION 28 : A B
QUESTION 29 : A B
QUESTION 30 : A B C